ACADEMIA PRO INTERLINGUA

DIRECTORE: PROF. G. PEANO
CAVORETTO - TORINO

a. 1921 d. 199.

THESAURARIO: PROF. G. PAGLIERO
VIA S. FRANCESCO 44 TORINO

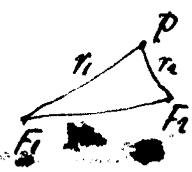
Honorato Collega,

Jam es longo tempore, que me recipe epistola ab Vos, de die 67. Me es multo occupato in schola, et ad me defice tempore pro responde prius.

Me attende articulo super occultatione de Venere per Luna, si pote perveni in tempore pro suo publicatione! In omni casu, alio articulo super alio quaestione es grato.

Problema super loco de punctos de flexu de curvas $r_1^m r_2^n =$ constante, es interessante.

Si u et v es vectore, $u \times v = \text{mod}\, u \times \text{mod}\, v \times \cos(u, v)$, producto interno (vide libros de Burali, Boggio, Marcolongo, Levi-Civita, Vivanti, etc).



Tunc $r_1^2 = (P-F_1)^2$, $r_2^2 = (P-F_2)^2$.

$r_1 dr_1 = (P-F_1) \times dP$.

Me differentia aequatione de curvas: $m r_2^2 (P-F_1) \times dP + n r_1^2 \cancel{(P-F_2) \times dP} = 0$ (1)

(dic que tangente dP es normale ad vectore $m r_2^2 (P-F_1) + n r_1^2 (P-F_2)$)

et me differentia uno secundo vice: $m r_2^2 \times (dP)^2 + 2m [(P-F_2) \times dP][(P-F_1) \times dP] + m r_2^2 (P-F_1) \ldots$
$+ n r_1^2 \times (dP)^2 + 2n [(P-F_1) \times dP][(P-F_2) \times dP] + n r_1^2 (P-F_2) \ldots = 0$

Isto formula determina $[m r_2^2 (P-F_1) + n r_1^2 (P-F_2)] \times d^2P$, ergo componente normale de d^2P, sed in punctos de flexu, isto componente $= 0$; ergo aequatione de punctos de flexu es:

$$(m r_2^2 + n r_1^2) \times (dP)^2 + 2(m+n)[(P-F_1) \times dP][(P-F_2) \times dP] = 0. \quad (2)$$

Me elimina dP inter aequationes (1) et (2). Si i es rotatione de angulo recto (quadrante recto), tunc (1) es satisfacto per $dP = i [m r_2^2 (P-F_1) + n r_1^2 (P-F_2)]$

Substitue in (2). $(dP)^2 = m^2 r_2^4 r_1^2 + n^2 r_1^4 r_2^2 + 2mn r_1^2 r_2^2 (P-F_1) \times (P-F_2)$

$= r_1^2 r_2^2 (m^2 r_2^2 + n^2 r_1^2 + 2mn (P-F_1) \times (P-F_2))$

$= r_1^2 r_2^2 [m(P-F_2) + n(P-F_1)]^2$

$= r_1^2 r_2^2 [(m+n)P - mF_2 - nF_1]^2 = r_1^2 r_2^2 \left[P - \frac{mF_2 + nF_1}{m+n}\right]^2$

Pone $F = \frac{mF_2 + nF_1}{m+n}$; et [illegible] pro origine. Pone ~~[illegible]~~ secundo F.

$F_2 - F_1 = 2au.$ $\quad (m+n)F_2 = (m+n)F + n2au.$ $\quad F_2 = F + \frac{n}{m+n} 2au$

$mF_2 + nF_1 = (m+n)F.$ $\quad (m+n)F_1 = (m+n)F - 2aum$, $F_1 = F - \frac{m}{m+n} 2au$

$P = F + re^{ix}u.$ $\quad (P-F)^2 = r^2,$ $\quad dP = i\left(mr_2^2\left(re^{ix}u + \frac{m}{m+n}2au\right) + nr_1^2\left(re^{ix}u - \frac{F}{m+n}\right.\right.$

$(P-F_1) \times dP = nr_1^2 (P-F_1) \times i(P-F_2)$

$(P-F_2) \times dP = -mr_2^2 (P-F_2) \times i(P-F_2)$

$dP = i\left[(mr_2^2 + nr_1^2) re^{ix}u + \frac{m^2r_2^2 - n^2r_1^2}{m+n} 2au\right]$

$(mr_2^2 + nr_1^2)\, (P-F)^2 \cancel{\frac{r_1^2 r_2^2}{[illegible]}} = 2\frac{\cancel{[illegible]}\, mn}{(m+n)}\, r_1^2 r_2^2 \left[(P-F_1) \times i(P-F_2)\right]^2$

~~$(mr_2^2 + nr_1^2)$~~

$P - F_1 = \left(re^{ix} + \frac{m}{m+n} 2a\right)u$

$P - F_2 = \left(re^{ix} - \frac{n}{m+n} 2a\right)u$

$\} \; m\left[r^2 + \frac{2n}{m+n} 2a\cos x + \left(\frac{n^2}{m+n}2\right)^2\right]$

~~$n[r^2 - \frac{2m}{m+n} \ldots (\frac{m}{m+n} 2)^2]$~~ $\}\, r_2^2 = \frac{2m}{m+n}\left[2\text{ area de triangulo } F_1 F_2 P\right]^2$

$\frac{2mn}{m+n}\left[2a \times r \sin x\right]^2$

$(m+n)r^2 + \frac{mn^2 + m^2n}{(m+n)^2} 4a^2 = \frac{2mn}{m+n} 4a^2 (\sin x)^2$

$(m+n) r^2 = \frac{4a^2 mn}{m+n}\left(2(\sin x)^2 - 1\right)$

~~$(m+n)$~~ $r^2 + \frac{4mn\, a^2}{(m+n)^2} a^2 \cos 2x = 0.$