In un' opera manoscritta (Fukudai-wo-Kaisuru-hō) rivista nel 1683. Confronta:

T Hayashi, Tōkyō Sūgaku-ButurigaKKwai Kizi (1) (2) 5 (1910), p. 254 : tradotto in italiano da S. Cherubino; Il Giornale di Matematiche (3) 3 (1912), p. 193.

D.E. Smith and Y. Mikami. A history of japanese Mathematics, Leipzig 1914, p. 124.

One of the most marked proofs of Seki's genius is seen in his anticipation of the notion of determinants. The school of Seki offered in succession five diplomas, representing various degrees of efficiency. The diploma of the third class was called the Fukudai-menkyo, and represented eighteen or nineteen subjects. The last of these subjects related to the fukudai problems or problems involving determinants, and since it appears in a

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revision of 1683, its discovery antedates this year. Leibnitz (1646-1716), to whom the Western world generally assigns the first idea of determinants, simply asserted that in order that the equations

$$10+11x+12y=0 \quad 20+21x+22y=0 \quad 30+31x+32y=0$$

may have the same roots the expression

$$10.21.32-10.22.31-11.20.32+11.22.30+12.20.31-12.21.30$$

must vanish. On the other hand, Seki treats of n equations. While Leibnitz's discovery was made in 1693 and was not published until after his death, it is evident that Seki was not only the discoverer but that he had a much broader idea than that of his great German contemporary. To show the essential features of his method we may first suppose that we have two equations of the second degree:

$$ax^2+bx+c=0 \quad a'x^2+b'x+c'=0.$$

Eliminating x^2 we have

$$(a'b - ab')x + (a'c - ac') = 0$$

and eliminating the absolute term and suppressing the factor x we have

$$(ac' - a'c)x + (bc' - b'c) = 0$$

That is, we have two equations of the second degree and transform them into two equations of the first degree by ~~that~~ what the Japanese called the process of folding (tatamu). In the same way we may transform n equations of the n^{th} degree into n equations of the $n-1$ degree [called Kwanshiki (substitute equations)]. From these latter equations the wasanka (Follower of the wasan [native mathematics]) proceeded to eliminate the various powers of x. Since it was their custom to write only the coefficients, including all zero coefficients, ~~and~~ not to equate to zero, their array of coefficients formed in itself a determinant, although they did not look upon it as a special function of the coefficients. On this array Seki now proceeds to perform two operations, the san (to cut) and the chi (to manage). The san consisted in the removal of a constant literal factor in any row or column, exactly as we remove a factor from a determinant to day. If the array (our determinant) equalled zero, this factor was at once dropped. The chi was the same operation with respect to a numerical factor.

Seki also expands this array of coefficients, practically the determinant that is the eliminant of the equations.

In this expansion some of the products are positive and these are called "sei" (kept alive), while others are negative and are called "koku" (put to death), and rules for determining these signs are given. Seki knew that the number of terms in the expansion of a determinant ~~are~~ of the n^{th} order was $n!$ and he also knew the law of ~~ti~~ interchange of columns and ~~of~~ rows. Whatever, therefore, may be our opinion as to Seki's originality in the "yenri"[1] or even as to his knowledge of that system at all or as to its value, we are compelled to recognize that to him rather than to Leibnitz is due the first step in the theory which afterwards, chiefly under the influence of Cramer (1750) and Cauchy (1812), was developed into the theory of determinants. The theory occupied the attention of members of the Seki school from time to time as several anonymous manuscripts assert, but the fact that nothing was printed leads to the belief that the process long remained a secret. It must be said, ~~however~~ however, that the Chinese and Japanese method of writing a set of simultaneous equations was such that it is rather remarkable that no predecessor of Seki's discovered the idea of the determinant.

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[1] The word may be translated "circle principle" or "circle theory", the name being derived from the fact that the mensuration of the circle is the first subject that it treats.